Lesson 27: Rotational Motion

Lesson Objectives:

- Students will calculate Torque in various situations.
- Students will apply the concept of inertia to rotational motion.
- Students will calculate angular velocity and momentum in various situations.

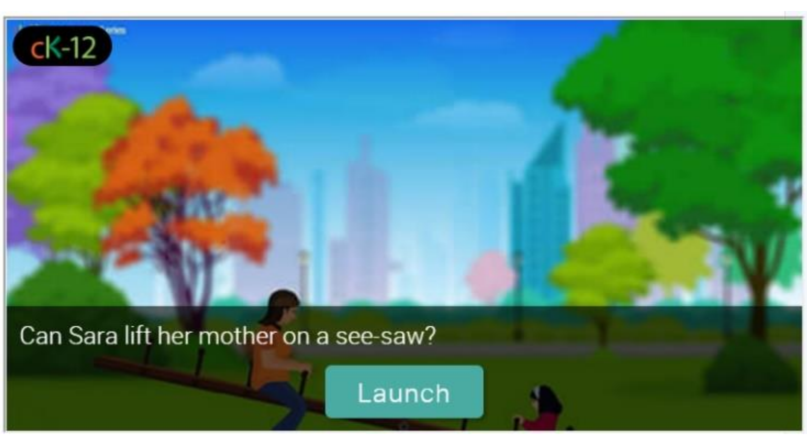
How Does a Seesaw Work?

It is easier to get a seesaw to move if you push on the board near the end rather than near the middle. This is due to torque - the rotational version of force.

The amount of force applied, as well as the location and the direction of the force with respect to the axis of rotation, determines the relative difficulty in causing a rotation. The longer the length of the seesaw, the easier it is to lift a heavy object.

We call the arm a **lever arm** or a **moment arm**. The component of a force applied perpendicular to a lever arm produces a **torque**. The torque is the product of the length of the lever arm and the force applied perpendicular to the lever arm.

In the **Figure** above the force F is applying a torque. For our purposes, we will consider torque to be a scalar quantity possessing clockwise and counterclockwise directions. Use the seesaw simulation below to further explore the concept of torque:



The Direction of Torque

We define the direction of the torque by noting clockwise (CW) and counterclockwise (CCW) motion of an object as a result of an applied force. Whether the object actually rotates or not is unimportant. We ask how the object *would* move were it free to do so. For example, in the **Figure** above the force *F* would rotate the meter stick in a counterclockwise direction. This is the same direction we turn a jar lid in order to loosen it.

Example 1

If a force F^1 was applied parallel to *F*, but to the left of *P* (see **Figure** above), in what direction would the meter stick turn?

Answer: The meter stick would turn clockwise.

You may be familiar with the expression: “Righty tighty, lefty loosey.”

We define the counterclockwise direction as positive and the clockwise direction as negative. The sign of the direction is based upon the Right Hand Rule. To understand this rule, hold your right hand with your thumb pointed up and curl your fingers into a fist. Notice that the direction your fingers curl in is counterclockwise (you’re looking down). We define the upward direction in which the thumb points as positive, and the corresponding counterclockwise torque as positive. If you turn your hand such that your thumb now points down and curl your fingers into a fist, you’ll see your fingers turn clockwise. We define the downward direction in which your thumb points as negative, and the corresponding clockwise torque as negative. (For those readers who have grown up using only digital clocks, the term clockwise originated from the rotational direction that the hands of an analog clock move; counterclockwise being the reverse rotational direction.)

Mathematical Definition of Torque

We can state the magnitude of the torque in two ways:

(1) The product of the perpendicular distance from the axis of rotation *r* (to the applied force) and the perpendicular component of the force $F\sin\Theta$.

(2) The product of the perpendicular distance $r\sin\Theta$ to the direction in which the force acts, and F.

Both (1) and (2) are equivalent to the product of r, F, and the sine of the angle between them. The symbol for torque is the Greek letter tau-- τ. Thus, we can write $\tau = rF\sin\Theta$, where the angle Θ is the angle between vectors*r* and *F*. If the angle between *r*and *F* is 90° then the torque is simply $\tau = rF$. We can see by the definition of the torque that the units of torque are *m*N* though they are usually expressed as *N*m* but never as joules even though they are dimensionally equivalent.

Example 2

An engineer is trying to turn a difficult nut. Using a long wrench, he applies a 105 N force at a distance 30.0-cm from the nut shown in **Figure 2** below. What is the magnitude of the torque applied by the plumber?

Answer: The lever arm is 30 cm and the angle between the force and the lever arm is 90-degrees.

$$\tau = Fr\sin\theta = (105\text{N})(0.30\text{m})\sin 90^\circ \rightarrow 31.5\ \text{N}\cdot\text{m}$$

[Figure 2]

Example 3

If the minimum torque required turning the pipe in **Figure 2** above is 31.5 N·m,, could a force smaller than 105 N be used?

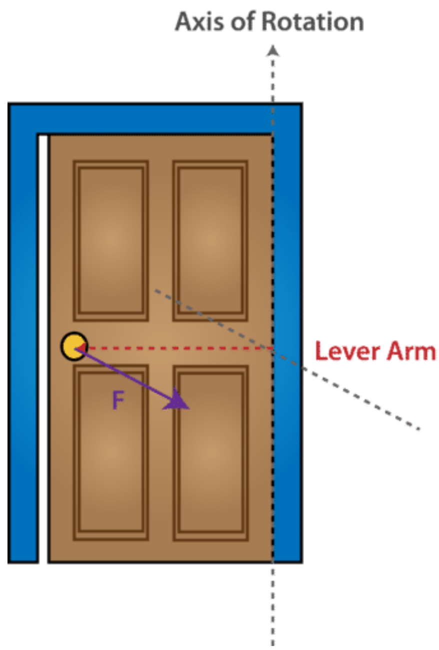
Answer: Yes. The workman could extend the length of the handle of the tool he's using to turn the pipe. The longer the arm of the tool, the smaller the force required. You have probably experienced this phenomenon yourself. If you have ever had a problem turning a screw, increasing the thickness of the screw driver handle enables you to provide the same torque with a smaller force. The fatter the handle, the farther your hand is from the axis of rotation, and so the smaller the force needed to turn the screw. This is how a lever works. For example, the longer the lever arm on a bottle opener, the smaller the force needed to pry the bottle cap open. You may have heard the famous dictum of Archimedes: "Give me a place to stand on and with a lever I will move the world." In other words, with a long enough lever arm, even the weakest person can move a tremendous weight.

Example 4

A door of width 0.810 m requires a minimum torque of *14.47 m*N* in order to open. The door knob is positioned 5.70 cm from the left edge of the door.

(a) What minimum force is required to open the door?

(b) If the doorknob is moved to the center of the door, what is the minimum force required in opening the door? Assume that the force acts perpendicular to the plane of the door. See **Figure 3** below.



[Figure 3]

Answers:

(a.) The force is perpendicular to the lever arm so τ = *rF*=14.47 m*N. Since the doorknob is 5.7 cm from the left edge of the door, the distance from the axis of rotation 5.7 cm from the left edge of the door, the distance from the axis is

$r = 0.81m - 0.057m = 0.753m$. The force required is then:

$$F = \frac{14.47\ m*N}{0.753\ m} = 19.216 \rightarrow 19.2\ N$$

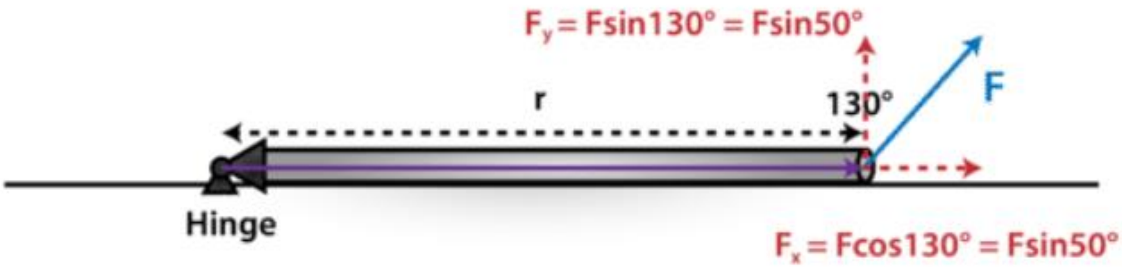
(b.) Since the doorknob is now in the center of the door, it is $\frac{0.81\ m}{2} = 0.405\ m$ from the axis of rotation. Therefore, the force required is

$$\frac{14.47\ m*N}{0.405\ m} = 35.73 \rightarrow 35.7\ N$$

How much force would be necessary to open the door if the doorknob were placed along the axis of rotation?

Example 5

In **Figure 4** below, a force *F* of 95.0 N is applied to a hinged rod of length *r* = 2.2 m. The angle between *F* and *r* is 130-degrees. Find the magnitude of the torque that the force exerts upon the rod.



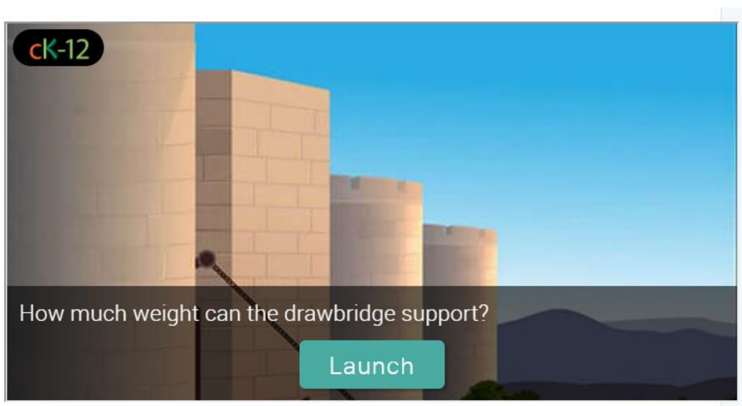
[Figure 4]

Answer:

Once again, $\tau = r(F\sin\theta)$, but this time since $\theta \neq 90^\circ$, the equation does not reduce to $\tau = rF$.

Therefore, $\tau = (2.20m)(95.0N)\sin 130^\circ = 160.1\ m * N$

The drawbridge in the simulation below has a cable that pulls on the bridge in order to rotate it counter-clockwise. Try to adjust the sliders so that all the forces in the bridge system are in equilibrium. Then, play around and see what conditions cause the bridge's cable to snap:



So, what makes an object more difficult to turn?

The difficulty it requires to push an object through space is called inertia or more precisely translational inertia. Inertia is equal to mass. The difficulty required to turn an object is called **rotational inertia** or sometimes “moment of inertia”. This is symbolized by the letter *I* (for inertia).

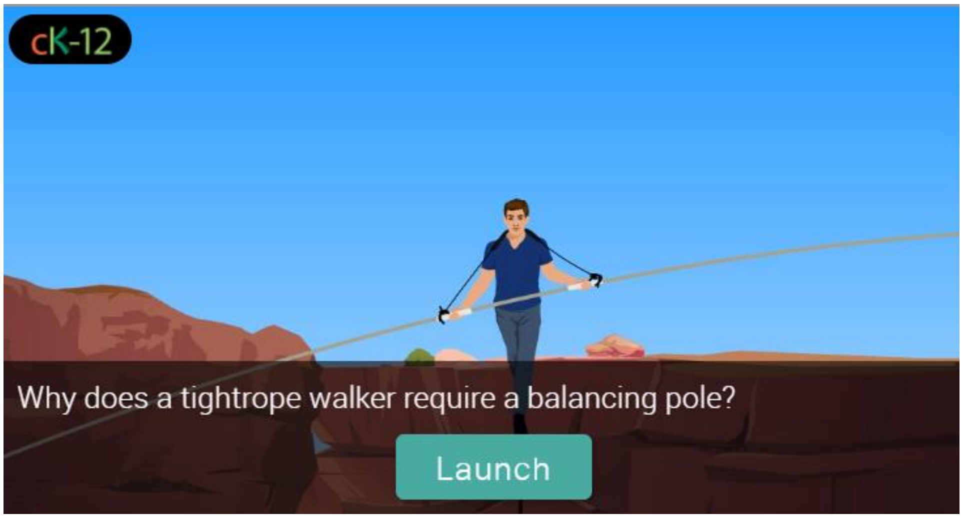
Try this out. Take a long object like a broomstick or baseball bat. Lay it flat and try to spin it with one hand. This can be difficult. Now instead, stand it upright and just give a twist with your fingers to turn it around. The same object is more difficult to spin one way than the other. Rotational inertia depends on both the mass and the mass distribution of an object. Mass closer to the axis is easier to turn. Mass farther from the axis is harder to turn.

Angular velocity is defined as how quickly an object is turning, and is symbolized by the Greek letter omega: ω. In physics, angular velocity is generally measured in one of two units:

- Revolutions per second, or rev/s. A complete rotation or revolution is equivalent to motion through 360-degrees. An object that turns around 30 times in one minute has an angular velocity of 0.5 rev/s.

- Radians per second or rad/s. A **radian** is the distance around the edge of a circle of radius 1. It takes 2π radians to complete one circle, so 2π—radians are equivalent to 1 revolution (360 degrees).

Use the simulation below to learn more about how a balancing pole can increase the rotational inertia of a tightrope walker and decrease his angular acceleration around the rope:



Linear momentum is defined as the product of mass and linear velocity ($p = mv$). In the same way, **angular momentum** is defined as the product of rotational inertia and angular velocity. The formula for angular momentum is stated as:

$L = I\omega$

where *I* is the rotational inertia (a term related to the distribution of mass) and the Greek letter omega ω is the angular velocity. Just like momentum in a given direction, objects undergoing rotation obey a similar conservation principle called conservation of angular momentum, which can be expressed as

$I_i\omega_i = I_f\omega_f$.

An important difference is that in linear momentum, the inertia is always the same. In angular momentum, the rotational inertia *I* and the angular velocity ω can change. Perhaps you've noticed that when a spinning figure skater pulls in her arms close to her body, her rotational velocity increases. Or perhaps you've seen a high driver spring off the diving board, tuck his legs close to his body, and spin quickly. What's going on? In each case the person brings more of their mass closer to the axis about which their body spins. The result is that their angular velocity increases.

The conservation of angular momentum ensures that, should the mass in the system move closer to the axis of rotation, the system will spin (rotate) more quickly. A classic demonstration of the conservation of angular momentum is shown in the following video. As the student in the figure moves the weights inward toward his body, his angular velocity increases, but his angular momentum stays constant.

Assignment

1. Why do we not calculate the torque using the cosine of the angle between rr"> and F?

2. Mathematically, the definition of torque can also be expressed as $\tau = (r \sin \Theta)\mathrm{F}$ In other words, we can always assume that F, rather than a component of the force, is responsible for rotation. How can this be?

3. A force of 5 N is applied to the end of a a lever that has a length of 2 meters. If the force is applied directly perpendicular to the lever, calculate the magnitude of the torque acting on the lever

4. A force of 10 N is applied perpendicular to the end of a bar of length 0.2 m. Calculate the torque produced by the force.

5. Marcus is on a merry-go-round holding a string. You are standing next to him and holding the other end of the string so that the string forms a tangent line to the edge of the merry-go-round. If Marcus is 2 meters from the center of the ride, what torque are you applying on him if you pull with 30 N?

6. You have two coins; one is a standard U.S. quarter, and the other is a coin of equal mass and size, but with a hole cut out of the center.

   a. Which coin has a higher moment of inertia?
   b. Which coin would have the greater angular momentum if they are both spun at the same angular velocity?

7. A weight tied to a rope is swung in a circle. The weight has a constant speed of 5 m/s and the length of the rope is 1 m. What is the weight's angular momentum?

8. A star is rotating with a period of 10.0 days. It collapses with no loss in mass to a white dwarf with a radius of 0.001 of its original radius.

   a. What is its initial angular velocity?
   b. What is its angular velocity after collapse?

9. A merry-go-round consists of a uniform solid disc of 225 kg225kg"> and a radius of 6.0 m6.0m">. A single 80kg"> person stands on the edge when it is coasting at 0.200.20"> revolutions per sec. How fast would the device be rotating after the person has walked 3.5 m toward the center. (The moments of inertia of compound objects add.)$0.30 \mathrm{rev/sec}$

Explore More

10. The system pictured in the video above (which includes the student, weights, and spinning seat) has an initial rotational inertia I_i and an initial angular velocity ω_i 2.00 rev/s. After the student pulls the weights toward his chest, the final rotational inertia of the system is only 80% of its initial rotational inertia- that is 0.800 I_i. Assuming that the angular momentum of the system is conserved, what is the final angular velocity of the system?