Lesson 22: The Big Picture: Momentum

Lesson Objectives:

- Students will investigate and calculate the work, energy, power, mechanical advantage, and efficiency using simple machines involving linear and rotational motion.
- Students will apply the concepts of potential and kinetic energy to final velocity of an object-independent of path; evaluate the conservation of energy and momentum in simple harmonic motion.
- Student will learn and apply the definition of momentum, impulse, and Power

The Problem

(1.1) Consider the problem of the motion of a jet plane. The jet is traveling at a tremendous speed and uses a huge amount of fuel as it moves. Looking at the defintion of force developed as Newton's Second Law, F=ma, how would you calculate the acceleration in this case? What problem would there be using this formula?

(1.2) The problem is that the mass of the fuel is being consumed as the plane moves--so the mass is constantly changing. A balloon rocket models this same problem. As the air moves out of the balloon, the balloon accelerates more slowly at first, and then quicker as the balloon loses air. The air is the fuel for the rocket, but it also has mass and, therefore, has to be accelerated along with the balloon. Newton's Second Law as commonly written makes the assumption that mass is constant, which is rarely true.

(1.3) Isaac Newton didn't have this problem with his original statement of the law. Instead of talking about mass and acceleration separately, he used a quantity called momentum.

Definition of Momentum

(2.1) Momentum is the product of mass and velocity. We can think of momentum as being the measure of how difficult it is to stop an object that is in motion. It is twice as hard to stop an object that has twice the mass or twice the speed. Direction is important because you can't stop an object by pushing it from the side--all you can do is change its direction.

(2.2) The importance of using momentum to describe motion is that the effect of forces can be calculated on the basis of changing mass and changing acceleration.

(2.3) Momentum is shown by the equation is p = mv, where p is the momentum of the object. Think about what this means when compared with Newton's second law...remember that acceleration is simply a change in velocity over a certain amount of time.

Impulse

(3.1) If a force causes acceleration of an object, then this creates a changed in velocity, which will in turn change the momentum of the object (provided the mass remains constant). A change in momentum is called an **impulse**, and it is written as Î”p or J. By examing the following algebraic manipulation:

Since, p_2=mv_2 and p_1=mv_1

then, Î”p = m(v_2 - v_1)

which is the same as saying **Î”p = mÎ”v**

and since the change in velocity over a change in time is acceleration, we can get

Î”p / Î”t = mÎ”v / Î”t = ma = F

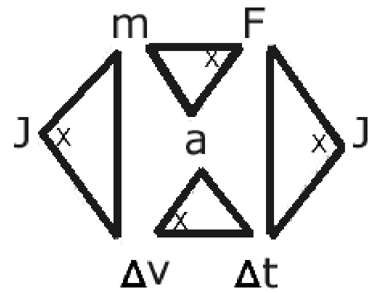
so Î”p / Î”t = F

or **Î”p = FÎ”t**

(3.2) This leads to the Impulse-Momentum Theorem, which states that force is the rate of change of momentum or Î”p = FÎ”t. If mass is constant, this equation can be rewritten as mÎ”v = FÎ”t, which is comparable to Newton's Second Law. In solving problems, the entire expression can be written as:

J = FÎ”t= mÎ”v = Î´p

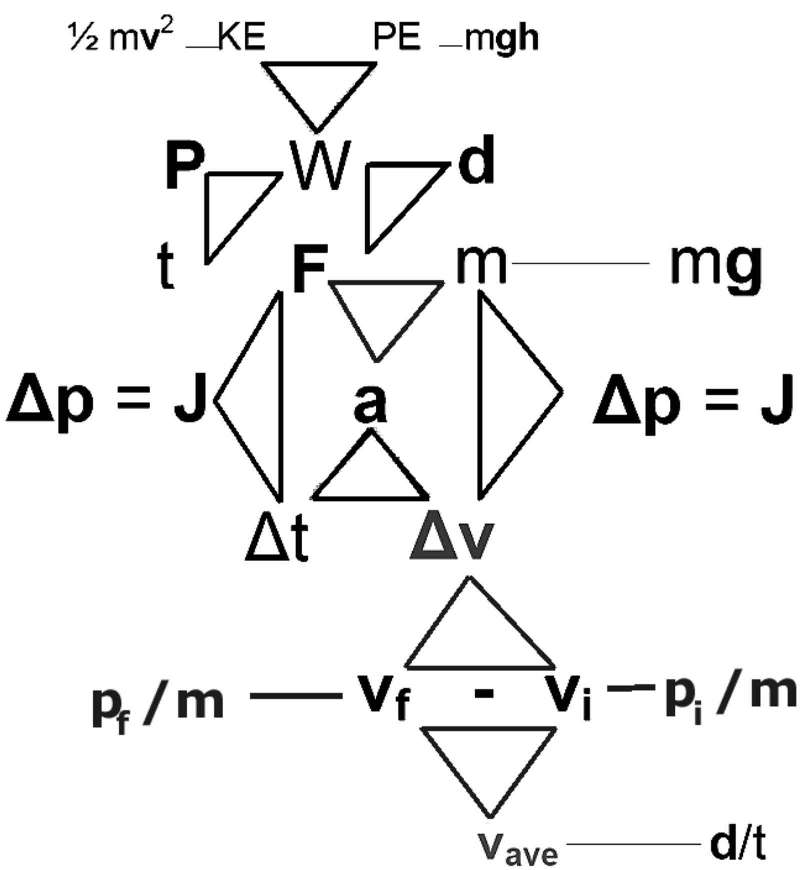
(3.3) This can be written graphically as:



The x notes which term is the product of the other two.

(3.4) **Power** is the rate at which work is done. Intuitively, we recognize power in our everyday lives. if someone were to lift a very large rock and it took them ten seconds, then we know that they showed some degree of power. But if they lifted the same rock in one second, then that would be a greater display of power. The same amount of work was done in each case, but the greater power was demonstrated by doing it in a shorter period of time. Power, algebraically, is the work divided by the time it took to do the work. P=W/t. Remember not to confuse the big variable P for power with the small variable p for impulse.

(3.5) Most problems will rely on you to be able to find the quantities you need through the relationships we've seen. This diagram will help you to see the relationships you'll need to solve the problems. Note that you still need to know how the items are related (whether it's a sum (+) or product (x) relationship). Remember a Newton is the amount of force required to accelerate a mass of one kilogram at a rate of one meter per second squared. So 1N = 1 kg*$^{m}/_{s}{}^{2}$.



J = FÎ”t = mÎ”v = Î”p

$p_i = mv_i$ and $p_f = mv_f$

$v_f = v_i + at$

$d = v_it + {}^1/_2\,at^2$ and $d = v_ft - {}^1/_2\,at^2$

$d = {}^1/_2(v_i + v_f)t$

$v_f^2 = v_i^2 + 2ad$

$v_{ave} = {}^1/_2(v_i + v_f) = {}^d/_t$

$KE = {}^1/_2mv^2$ and $PE = mgh$

W = Î”E and mechanical $W = Fd$

$P = W/t$

Grading for this lesson:

- **10:** Your answers need to be correct and you can have no grammatical errors, within the second revision of this lesson. Answers are correct, complete, and clear; all lesson requirements have been met.
- **9:** Your answers need to be correct and you can have no grammatical errors, within the fourth revision of this lesson.
- **8:** You can have up to 1 error after any number of revisions.
- **7:** This grade will be given when the student is not paying attention to instruction, for lack of effort on the part of the student, or for rude or inappropriate responses. All revisions must still be completed. Repeated offenses may lead to an even lower grade.
- **6:** Reserved for administrative use.
- **5:** Plagiarism â€“ purposeful or mistaken which will lower your final grade for the course (so be very careful when posting your work!); lack of effort, disrespect, or attitude (we are here to communicate with you if you donâ€™t understand something); or 9 or more errors; or lesson requirements have not been met.

Also be aware that you will have a chance to revise your work. More than two revisions will result in a lower grade, so read the directions carefully and make sure you meet the requirements.

No lesson is complete without the approval of the instructor, and all revisions must be completed before a grade is assigned. No grade will be given for incomplete work.

Assignment

1. A hockey player hits a puck weighing 0.25kg with a +55 N force. If the hockey stick is in contact with the puck for .1 s and a displacement of +0.80m, find the:

Hint: Don't do (a) first! You'll need to do (b) to find the force, then (d) to find the Work in order to do (a) and (c)....

(a) power dissipated (hint, use P = W/t)

(b) puck's acceleration (hint, use a = F/m)

(c) impulse (hint, use J = Ft)

(d) work done (hint, use W = Fd)

2. A 1200kg car slows down using an average braking force of 1400N over a time interval of 10s. If the distance it travels while slowing down is 120m:

(a) What impulse does the car experience?

(b) What is the car's acceleration?

(c) How much work has been done on the car?

(d) How much power is used in stopping the car?

3. A soccer ball with a mass of 0.42 kg is accelerated at a rate of +800.0 $^{m}/_{s}{}^{2}$over a time interval of 0.010s by a soccer player's kick. If her foot is in contact with the ball for a distance of +1.0cm find:

(a) the force the soccer player exerts on the ball.

(b) the impulse applied to the ball.

(c) the work done by the kicker on the ball.

(d) the power of the player's kick.

4. A 0.14 kg baseball is pitched at a speed of 35 m/s. If it is hit straight back at the pitcher with a force of 54000N exerted over a time interval of 1.0 millisecond, find:

(a) the acceleration of the baseball (hint, use a = F/m)

(b) the impulse given to the baseball (hint, use J = Ft)

(c) the change in momentum of the baseball (hint, Î”p = J)

(d) the initial momentum of the baseball (hint, p_i = mv)

(e) the final momentum of the baseball (hint, p_f - p_i = Î”p, so p_f = Î”p + p_i

(f) the final velocity of the baseball (hint, use v_f = ^{p_f} / $_m$)

(g) the distance that the baseball was in contact with the bat (hint, use d = v_{ave}t)

5. A ball falls for 9.0s increasing its kinetic energy by 2700 J. If the force acting on the ball is 6.0 N find the following quantities:

(a) change in potential energy (hint, Î”GPE = Î”KE)

(b) work done by the ball (hint, W = Î”KE)

(c) distance the ball falls in 9.0s

(d) mass of the ball

(e) power dissipated by the fall

(f) impulse the ball experiences

(g) change in momentum

(h) change in velocity

(i) average velocity

(j) initial velocity

(k) final velocity

(l) initial momentum

(m) final momentum