Lesson 23: Area and Volume of Prisms and Pyramids

Lesson Objectives:

- Student will apply formulas to calculate area and volume of prisms and pyramids.
- Student will use the properties of special right triangles to calculate measurements needed for area and volume of prisms and pyramids.

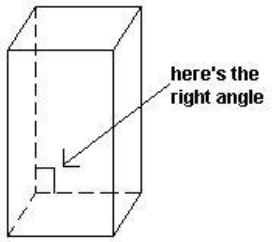
Now we'll look at some other shapes: prisms and pyramids.

There are formulas that deal with the surface area and volume of prisms, but only right prisms. In other words, only those which are based on regular shapes, and make a right angle from the base to the matching top. "Right prisms are prisms that have two special characteristics -- all lateral edges are perpendicular to the bases, and lateral faces are rectangular." If one wants to know the total surface area, you can just add the area of each surface (the top and bottom, and the sides). Often, however, the question will be asking you for the *lateral* area, or the area of the sides alone.

Right Prism Area Theorem

The area L of any right prism is equal to the product of the perimeter P of a base and the height h of the prism.

Area (lateral area, as opposed to total area) Formula: L = Ph



Right Prism Volume Postulate

The volume V of any right prism is the product of B (the area of the base) and the height h of the prism.

Volume Formula: V = Bh

So for the area, the problem could look like this:

What is the area of a prism with a height of 6 and base perimeter of 24?

L = Ph
L = 24(6)
L = 144

And that's it!

A possible volume problem would be:

What is the area of the prism if the area of one base is 36 and the height is 6?

V = Bh
V = 36(6)
V = 216

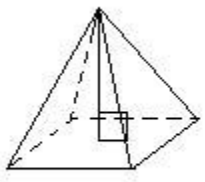
And that's it!

The only additional difficulty that might appear is that you'd have to determine the perimeter or the area of the base yourself. Remember that for squares and rectangles, the perimeter is the sum of the lengths of all the sides (or 2w + 2l), and the area is length multiplied by width.

Regular Pyramid Area Theorem

The area L of any regular pyramid with a base that has perimeter P and with slant height l is equal to one-half the product of the perimeter and the slant height.

Formula: L = (1/2)(P)l



Pyramid Volume Theorem

The volume V of any pyramid with height h and a base with area B is equal to one-third the product of the height and the area of the base.

Formula: V = (1/3)Bh

A pyramid is a polyhedron with a single base and lateral faces that are all triangular. All lateral edges of a pyramid meet at a single point, or vertex. A regular pyramid is a pyramid that has a base that is a regular polygon and with lateral faces that are all congruent isosceles triangles.

At any rate, the equations for area and volume are just like all the others we've done--just plug in the numbers.

If a regular pyramid has a square base with a length and width of 3, and slant height of 5, and a height of 4 then what is the area of the pyramid, and what is the pyramid's volume.

L = (1/2)(P)l
L = (1/2)(12)5
L = 30

V = (1/3)Bh
V = (1/3)9(4)
V = 12

Here is a video.

Video One

Practice

Click the link below to do some practice:

Solid geometry

Grading for this lesson:

To get a 10: All answers are correct the first time, or within first revision.
To get a 9: You can have 1 incorrect answer after your original submission.
To get an 8: You can have 2 incorrect answers after your original submission.
To get a 7: You can have 3 incorrect answers after your original submission.
To get a 6: You can have 4 incorrect answers after your original submission.
To get a 5: Cheating - Plagiarism - purposeful or mistaken, which will lower your final grade for the course (so be very careful when posting your work!); lack of effort, disrespect, or attitude (we are here to communicate with you if you don't understand something);

Note: For this class it is necessary to post the questions over each answer. Failure to do so will result in asking for a revision. **No grade will be given for incomplete work**.

Assignment:

If you get more than 3 wrong, you will be asked to resubmit the wrong answers and show your work. The teacher will then look at your work and give you advice on what you are doing wrong.

For 1 through 10, what is the area and volume of the given shape, if the length of one side of the base is 6, the height is 8, and the slant height is 10? (Not all shapes will require all three numbers.)

The shape is a right prism with:

1. an equilateral triangle as the base

2. a rectangular base with a width of 3

3. a square base

4. a rectangular base with a width of 5

5. the base is an isosceles triangle with a height of 8 and a base of 3 (sides of 6)

The shape is a pyramid with:

6. a rectangular base with a width of 4

7. a square base

8. a rectangular base with a width of 3

9. a rectangular base with a width of 5

10. a rectangular base with a width of 7

I have an isosceles triangle with a height of 4 and a base of 6:

11. What is the area?

This triangle just became the base of a regular prism, with a height of 8:

12. What is the lateral area?

13. What is the volume?

14. What is the area of the largest rectangular side?

I have a rectangle, with a length of 7 and a width of 4:

15. What is the perimeter?

16. What is the area?

This rectangle just became the base of a regular prism, with a height of 6:

17. What is the lateral area?

18. What is the total surface area?

19. What is the volume?

20. What is the area of the largest rectangular side?